A Constant-Space Algorithm for Leader Election in Partially Populated 2D Meshes

John S. Pezaris
MIT Room NE43-616
pz@mit.edu

1 The Problem

In a mesh-connected group of processors, there is often the need to individuate each processor by unique identification number or address. Automatic generation of these addresses for arbitrarily populated meshes requires a global reference point, or origin, from which addresses may be propagated. This paper presents an algorithm that selects an origin node for an arbitrarily-populated two-dimensional four-neighbor mesh using only local information and communication and runs in constant space per node.

2 Hypothesis Disproving

The hypothesis disproving algorithm has two phases: the first phase identifies potential, or hypothesized, origins; the second phase propagates messages from these nodes around the perimeter of the mesh, each asserting the hypothesis that its originating node is the mesh origin. As messages are received by each of the other hypothesized origins they are used to disprove conflicting hypotheses. At the termination of the algorithm, exactly \( n \) active hypotheses (each at distinct nodes) will remain, one for each isolated submesh; these hypotheses are verified as part of the termination process, and the associated nodes declared mesh origins.

3 The Algorithm

3.1 Phase One

The first phase passes in two steps. At the receipt of a global start signal, each node independently counts the number of neighbors it sees to determine if it is an interior or perimeter node: if it has four neighbors, it is an interior node, if fewer than four, a perimeter node. Then, each perimeter node determines if it is a hypothesized origin by the positioning of its neighbors: if it has two neighbors, one to the north and one to the east, or one neighbor, to the north or to the east, it is a hypothesized origin; otherwise it is a normal perimeter node. Interior nodes take no further part in the computation.

3.2 Phase Two

At the start of the second phase, each hypothesized origin sends a message to the east (or to the north, in the case of a single-neighbor node with a northward neighbor). The message contains three data (\( \Delta x, \Delta y, \Delta \phi \)) which keep track of the net distance the message has traveled in the \( x \) and \( y \) directions, and the total number of 90-degree left-hand turns. The first two form a localized relative address; the third is used to disambiguate between interior and exterior perimeters.

Perimeter nodes receive messages and deliver them according to a left-hand-to-the-wall rule, after updating the message fields. Hypothesized origins further use the message fields (which contain the relative position of the sending hypothesized origin node) to disprove either the hypothesis that the current node is an origin, or that the sending node is an origin. If the local hypothesis is disproved, the node degenerates to a normal perimeter node and the received message is propagated; if the message hypothesis is disproved, the message is destroyed. As messages are passed from

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node to node, they circumnavigate the mesh perimeter and return to their origination point, unless they are destroyed.

A message has returned to its originating node when it is delivered with a net distance of \((0,0,\pm 4)\). Because (1) no two nodes can occupy the same address, (2) all messages travel the same path and visit all hypothesized origin nodes, (3) messages are not delivered out of order, and (4) inconsistent hypothesis messages are destroyed, only one node per perimeter will receive the message it generated. This node will be declared the mesh origin, terminating the algorithm.

Should the mesh have holes, there are potentially additional interior perimeters which may contain false origins (see Figure 1). Interior origins are disproved in a final step by examining the net number of left-hand turns the message has taken: if \(\Delta \phi\) is positive \((4)\), then the message has traveled an exterior path, and the originating node is the mesh origin; if \(\Delta \phi\) is negative \((-4)\), then the originating node is on an interior path, and disproves itself.

4 Constant Space

This algorithm, as thus-far described, requires space per node which is proportional to the logarithm of the mesh diameter to encode the distance traveled by each message. The space requirements can be reduced to constant space per node by serially encoding the messages and distributing their storage around the perimeter path. The bulk of the operation is as presented above, save that the messages are encoded in three simultaneously-transmitted serial bitstreams (one for each field), and the comparison and updating operations are done serially.

4.1 Deadlock Avoidance

Deadlock is possible since message length grows with the diameter of the mesh and messages are asynchronously delivered along a closed path. By observing (1) the maximum fraction of hypothesized origins and therefore messages along the perimeter, and (2) the maximum message length, the number of bits transmitted per communication cycle (symbol complexity) can be set to prevent deadlock for all meshes. A formal proof is available.

5 Conclusion

A constant-space algorithm for selecting a single origin node from partially-populated four-neighbor two-dimensional mesh has been described. A functioning event-driven simulator has been written for the \(O(n)\) version of this algorithm; a constant-space version of the simulator is under development.

6 Bibliography


Figure 1: Snapshot of a partially-populated mesh with three hypothesized origins—two exterior (A and B), one interior (C)—and three in-flight messages, approximately two time steps after reset. The message \(1,1,0\) is from A; the message \(1,1,1\) is from B; and the message \(1,-2,-2\) is from C.